

3 Minimization of Boolean Algebra

3.1

3.2 2,3

3.3 4

3.4 5

3.5

3.6 NAND NOR

3.7 2

3.8

3.9

3.10

3.11

3.12

Karnaugh map

□

□ K-map

◆ ,

◆ 가

◆ 1, 2, 4, 8,, 2^n , 0, 1, 2, 3,, n literal

□ “1” f , “0” f

□ 가 7

2-variable map

		y	
		0	1
x	0	$x'y$ (m_0)	$x'y$ (m_1)
	1	xy' (m_2)	xy (m_3)

		y	
		0	1
x	0	0	0
	1	1	1

$$xy' + xy = x(y' + y) = x$$

		y	
		0	1
x	0	0	1
	1	0	1

$$x'y + xy = (x' + x)y = y$$

		y	
		0	1
x	0	1	1
	1	0	0

$$x'y' + x'y = x'(y' + y) = x'$$

		y	
		0	1
x	0	1	0
	1	1	0

$$x'y' + xy' = (x' + x)y' = y'$$

		y	
		0	1
x	0	1	1
	1	1	1

$$x + x' = 1 \text{ or } y + y' = 1$$

		y	
		0	1
x	0	0	0
	1	0	0

$$0$$

2-variable map

(ex)

x	y	f
0	0	0
0	1	1
1	0	1
1	1	1

(1)

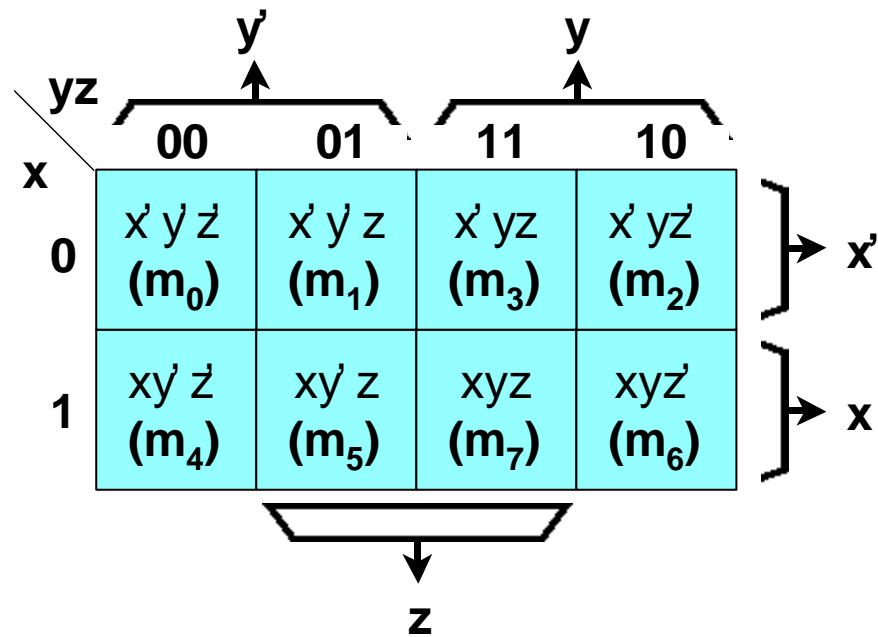
$$\begin{aligned}f &= x'y + xy' + xy \\ &= x'y + x(y' + y) \\ &= x'y + x \\ &= (x' + x)(y + x) \\ &= x + y\end{aligned}$$

(2) map

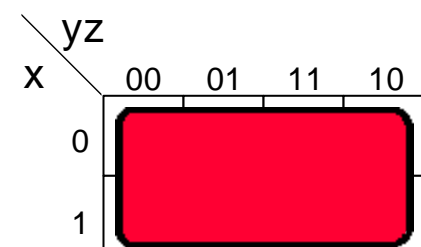
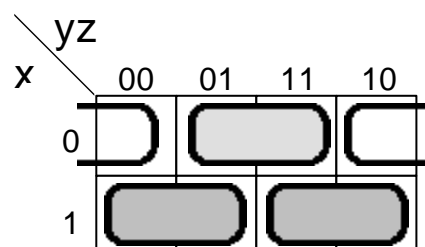
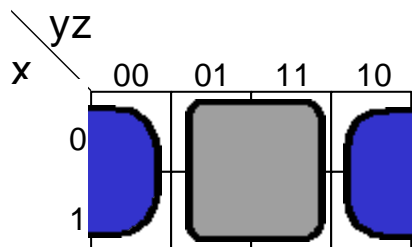
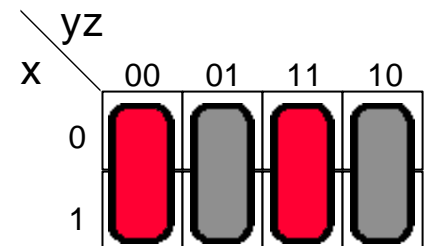
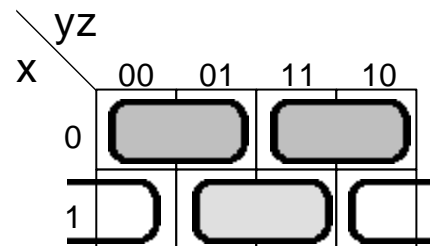
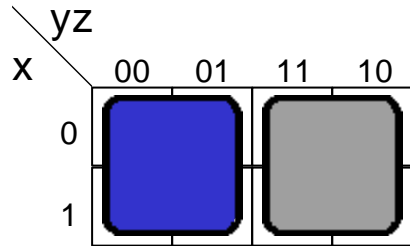
x \ y	0	1
0		1
1	1	1

$$f = x + y$$

3-variable map



3-variable map



3-variable map

(ex) (majority function)

“1”

가

“1”

x	y	z	f
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

$f = (3,5,6,7)$

		yz			
x		00	01	11	10
0				1	
1			1	1	1

$f = xy+yz+zx$

(ex) $f = (2,3,4,6,7)$

x	y	z	f
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

		yz			
x		00	01	11	10
0				1	1
1		1		1	1

$f = y+xz'$

3-variable map

$$f = A' C + A' B + AB' C + BC$$

(1) sum of minterm

(2) minimize f

		BC			
		00	01	11	10
A	0		1	1	1
	1		1	1	

$$f = (1,2,3,5,7)$$

$$= C + A' B$$

$$f = (1,2,4,7)$$

		BC			
		00	01	11	10
A	0		1		1
	1	1		1	

$$f = A' B' C + A' BC' + AB' C' + ABC$$

$$= A' (B' C + BC') + A (B' C' + BC)$$

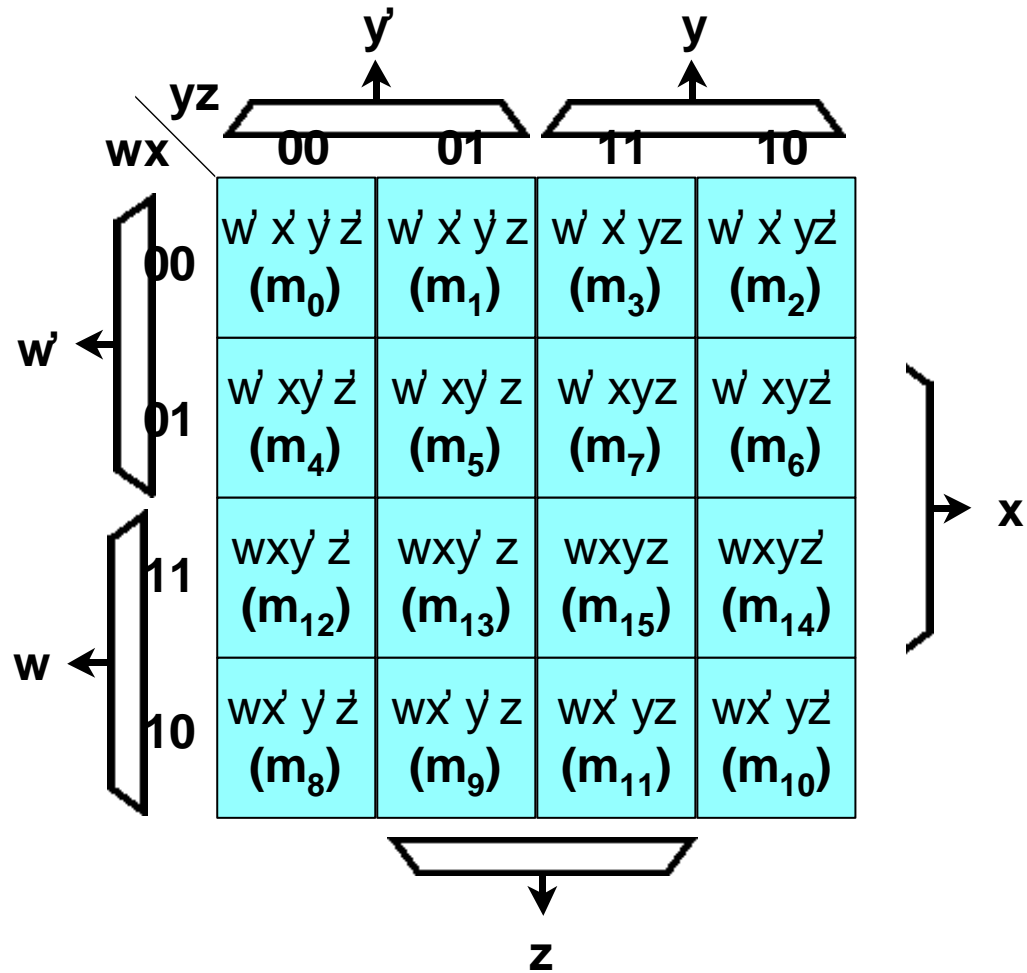
$$= A' (B \dot{A} C) + A (B \dot{C})$$

$$= A' (B \dot{A} C) + A (B \dot{A} C)'$$

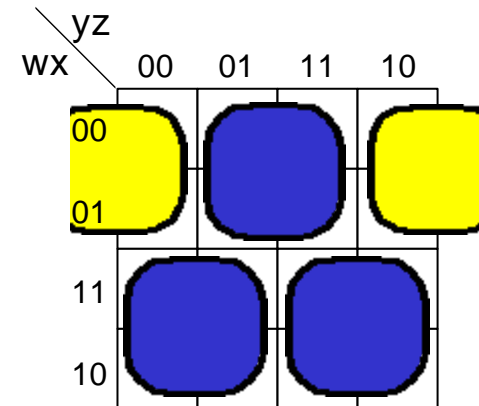
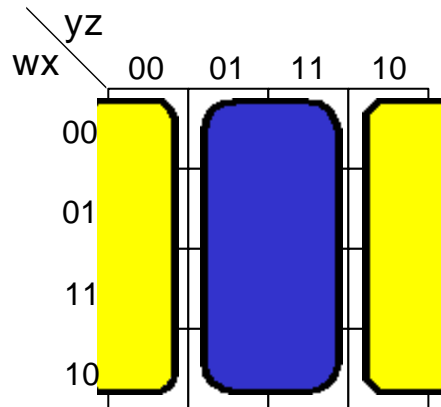
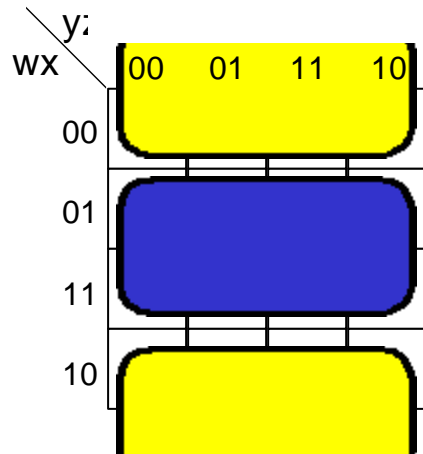
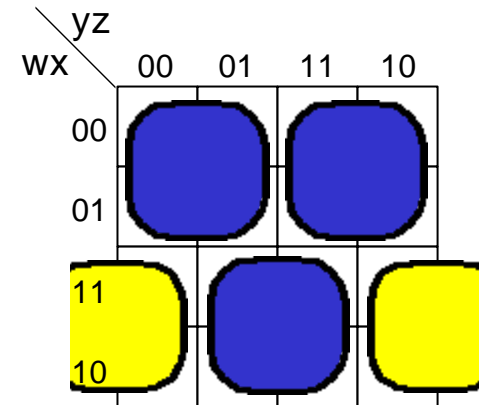
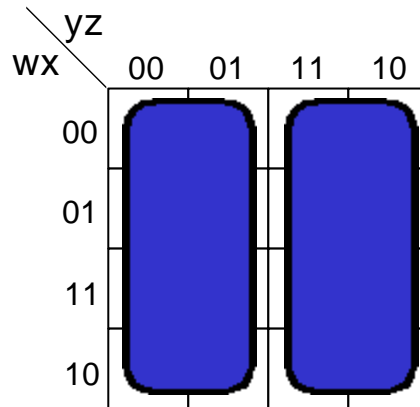
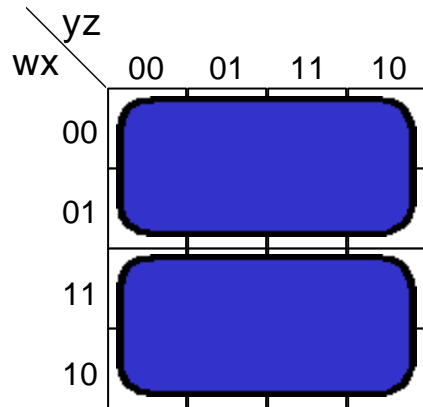
$$= A \dot{A} (B \dot{A} C)$$

$$= A \dot{A} B \dot{A} C$$

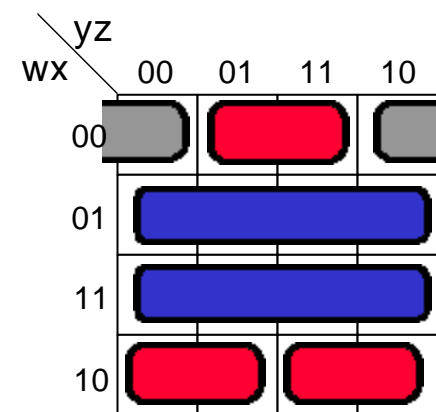
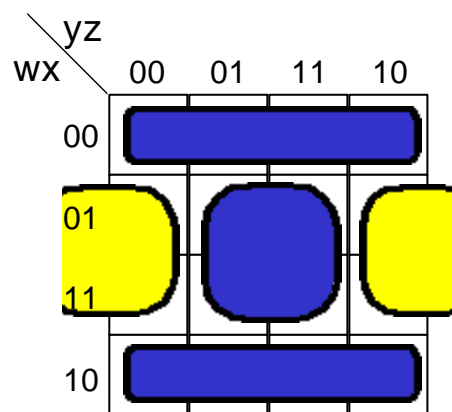
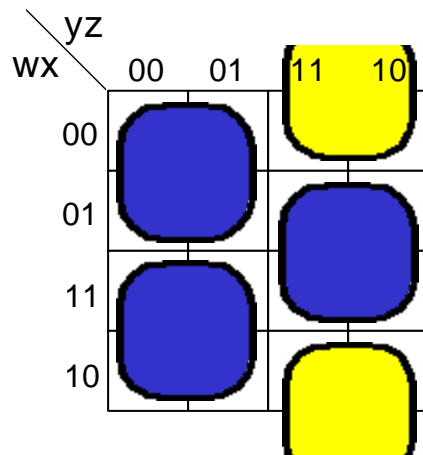
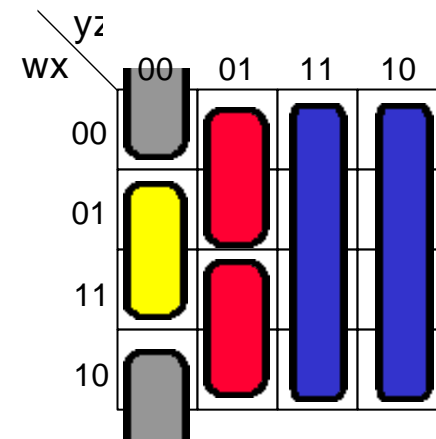
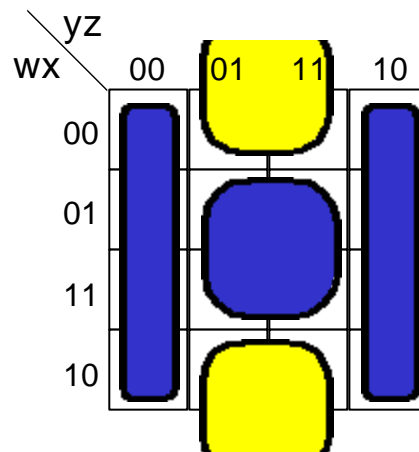
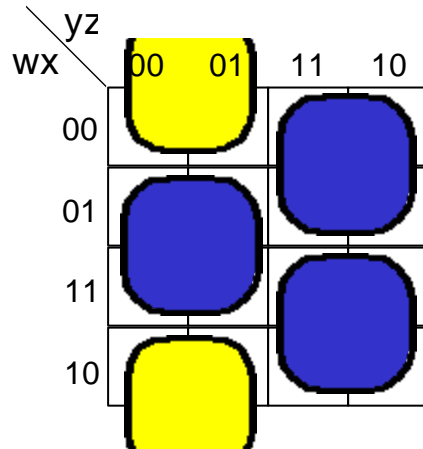
4-variable map



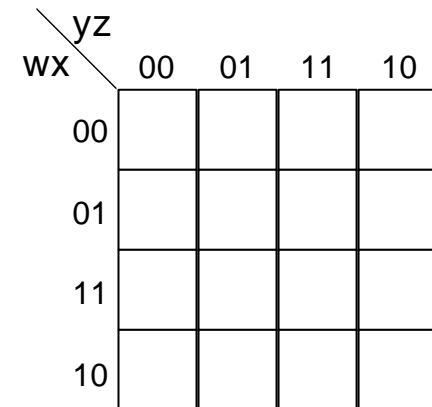
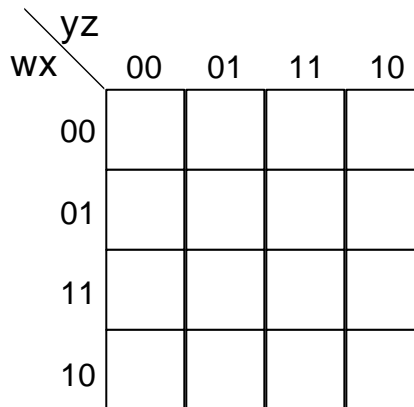
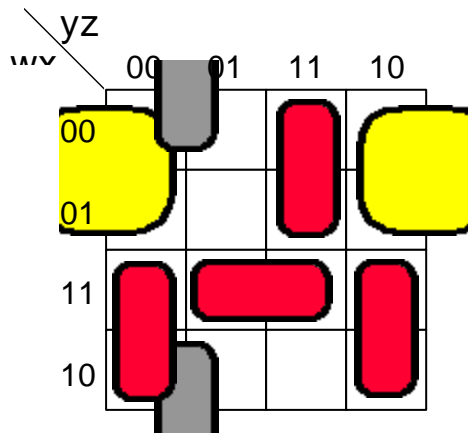
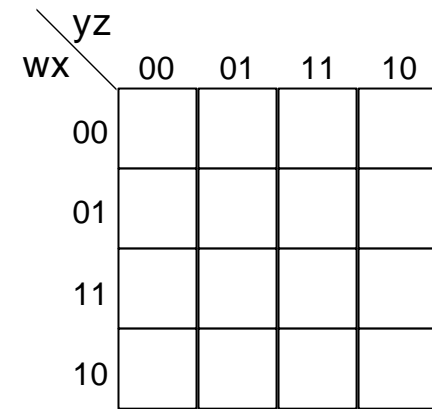
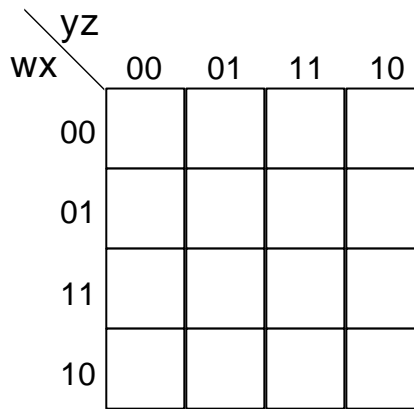
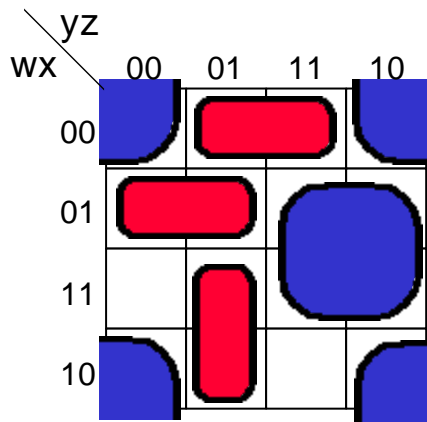
4-variable map



4-variable map



4-variable map



4-variable map

$$f = (0,1,2,4,5,6,8,9,12,13,14)$$

wx \ yz	00	01	11	10
00	1	1		1
01	1	1		1
11	1	1		1
10	1	1		

$$f = y' + w' z' + xz'$$

$$f = A' B' C' + B' CD' + A' BCD' + AB' C'$$

AB \ CD	00	01	11	10
00	1	1		1
01				1
11				
10	1	1		1

$$f = (0,1,2,6,8,9,10) \\ = B' D' + B' C' + A' CD'$$

4-variable map

$$f = (3,4,6,7,11,12,13,14,15) \longrightarrow f = (0,1,2,5,8,9,10)$$

	yz			
wx	00	01	11	10
00			0	
01	0		0	0
11	0	0	0	0
10			0	

	yz			
wx	00	01	11	10
00	1	1		1
01		1		
11				
10	1	1		1

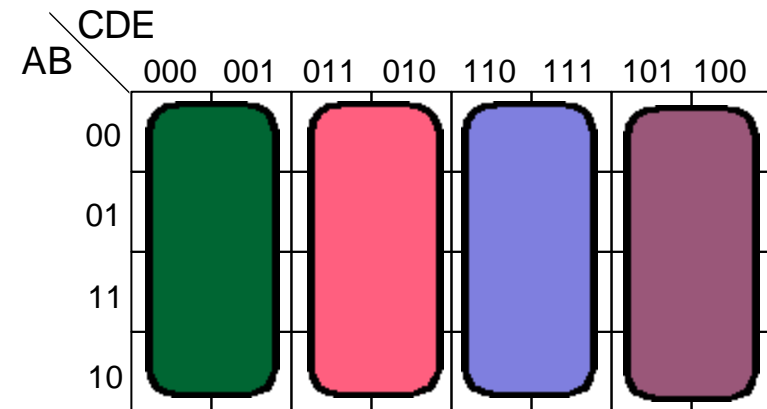
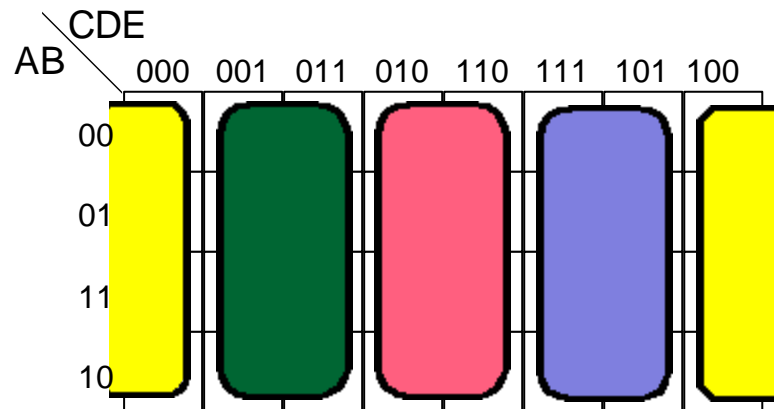
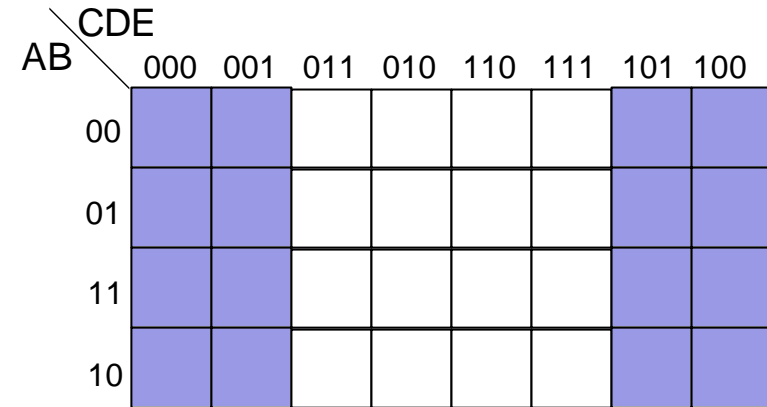
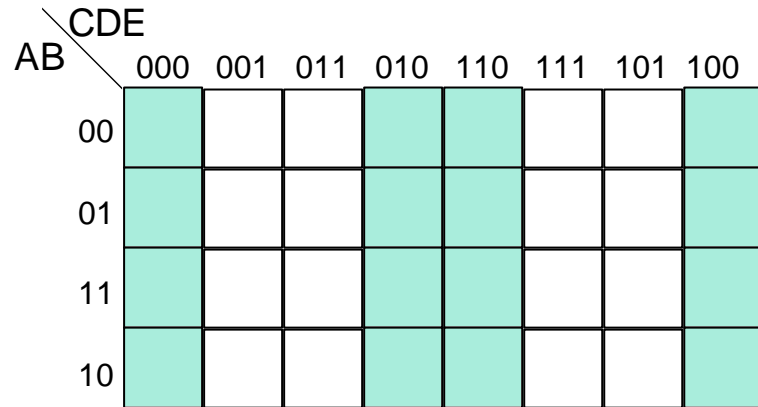
$$\begin{aligned} f' &= wx+yz+xz' \\ f &= (f')' \\ &= (wx+yz+xz')' \\ &= (w'+x')(y'+z')(x'+z') \end{aligned}$$

$$f = x'z' + x'y + w'yz$$

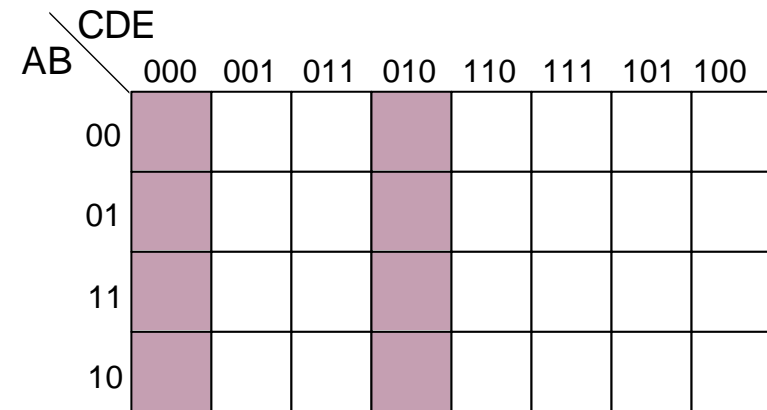
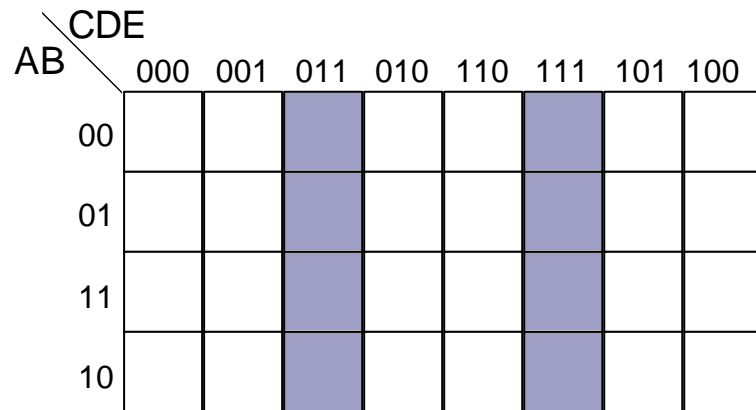
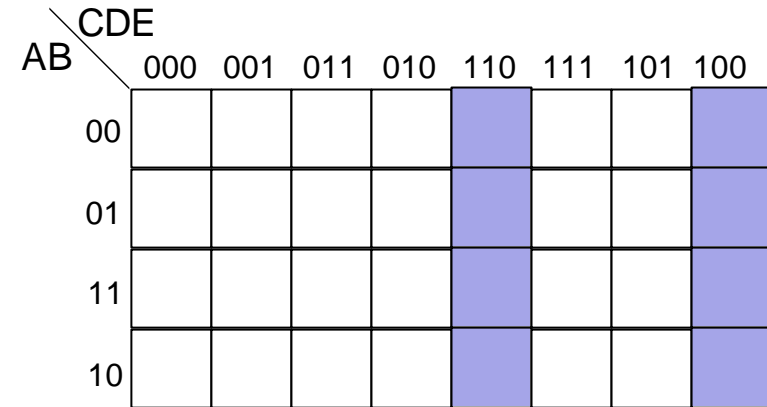
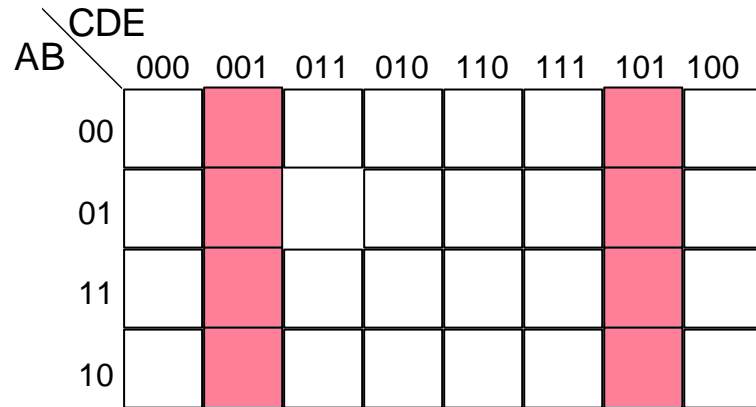
5-variable map

		CDE							
		000	001	011	010	110	111	101	100
AB	00	$A'B'C'DE$ (m_0)	$A'B'CD'E$ (m_1)	$A'BC'DE$ (m_3)	$A'BC'D'E$ (m_2)	$A'BCDE$ (m_6)	$A'BC'DE$ (m_7)	$A'BCD'E$ (m_5)	$A'BCD'E$ (m_4)
	01	$A'BC'D'E$ (m_8)	$A'BC'DE$ (m_9)	$A'BC'DE$ (m_{11})	$A'BC'D'E$ (m_{10})	$A'BCDE$ (m_{14})	$A'BCDE$ (m_{15})	$A'BCD'E$ (m_{13})	$A'BCD'E$ (m_{12})
	11	$ABC'D'E$ (m_{24})	$ABC'D'E$ (m_{25})	$ABC'DE$ (m_{27})	$ABC'D'E$ (m_{26})	$ABCDE$ (m_{30})	$ABCDE$ (m_{31})	$ABCD'E$ (m_{29})	$ABCD'E$ (m_{28})
	10	$AB'C'DE$ (m_{16})	$AB'C'D'E$ (m_{17})	$AB'CD'E$ (m_{19})	$AB'CD'E$ (m_{18})	$AB'CDE$ (m_{22})	$AB'CDE$ (m_{23})	$AB'CD'E$ (m_{21})	$AB'CD'E$ (m_{20})

5-variable map



5-variable map



5-variable map

$$f = (0,2,4,6,9,13,21,23,25,29,31)$$

AB \ CDE		CDE							
		000	001	011	010	110	111	101	100
00	0	0	1	3	2	6	7	5	4
	01	8	9	11	10	14	15	13	12
11	24	24	25	27	26	30	31	29	28
	10	16	17	19	18	22	23	21	20

AB \ CDE		CDE							
		000	001	011	010	110	111	101	100
00	0	1			1	1			1
	01		1					1	
11	24		1				1	1	
	10						1	1	

$$f = A' B' E + BD' E + ACE$$

5-variable map

$$f = (0,2,4,6,9,11,13,15,17,21,25,27,29,31)$$

AB \ CDE		CDE							
		000	001	011	010	110	111	101	100
00	0	0	1	3	2	6	7	5	4
	01	8	9	11	10	14	15	13	12
11	24	24	25	27	26	30	31	29	28
	10	16	17	19	18	22	23	21	20

AB \ CDE		CDE							
		000	001	011	010	110	111	101	100
00	0	1			1	1			1
	01		1	1			1	1	
11	24		1	1			1	1	
	10		1					1	

$$f = BE + AD'E + A'B'E$$

6-variable map

Product of Sum

- (1) "0" f sum of product
- (2) f f

(ex) f = (0,1,2,5,8,9,10)

	CD			
AB \	00	01	11	10
00	1	1		1
01		1		
11				
10	1	1		1



	CD			
AB \	00	01	11	10
00			0	
01	0		0	0
11	0	0	0	0
10			0	

sum of product

$$f = B' D' + B' C + A' C D$$

product of sum

$$f' = AB + CD + BD'$$

$$f = (f')$$

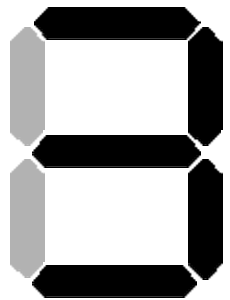
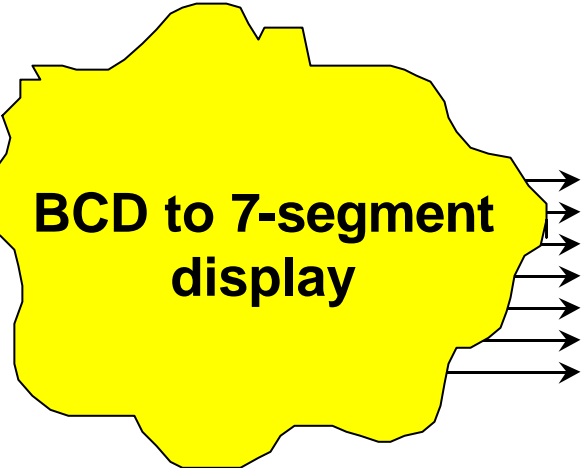
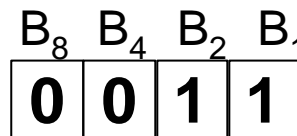
$$= (A' + B')(C + D)(B' + D)$$

Don't care condition(無關條件)

- ◆ minterm
- ◆ (don't care)

(ex) BCD to 7-segment display

B ₈	B ₄	B ₂	B ₁
0	0	0	0
0	0	0	1
0	0	1	0
0	0	1	1
0	1	0	0
0	1	0	1
0	1	1	0
0	1	1	1
1	0	0	0
1	0	0	1
1	0	1	0
1	0	1	1
1	1	0	0
1	1	0	1
1	1	1	0
1	1	1	1



Don't care condition

$$f(w,x,y,z) = (1,3,7,11,15)$$

$$d(w,x,y,z) = (0,2,5)$$

		yz			
	wx	00	01	11	10
00		X	1	1	X
01			X	1	
11				1	
10				1	

$$f = yz + w' x'$$

or

		yz			
	wx	00	01	11	10
00		X	1	1	X
01			X	1	
11				1	
10				1	

$$f = yz + w' z$$

Don't care condition

$$f(w,x,y,z) = (1,3,6,7,11,15)$$

$$d(w,x,y,z) = (4,5,14)$$

		yz			
wx		00	01	11	10
00			1	1	
01		X	X	1	1
11				1	X
10				1	

or

		yz			
wx		00	01	11	10
00			1	1	
01		X	X	1	1
11				1	X
10				1	

$$f = w'x + yz + w'z$$

$$f = yz + xy + w'z$$